Honors Solutions

Honors Lesson 1

1. Trigonometry
2. Arithmetic
3. Algebra
4. Calculus
5. Geometry

Honors Lesson 2

1. $Y^4 = 2X^2$
   Make Y negative:
   $(-Y)^4 = 2X^2$
   $Y^4 = 2X^2$
   Make X negative:
   $Y^4 = 2(-X)^2$
   $Y^4 = 2X^2$
   In both cases, changing the variable produces the original equation, so this equation is symmetrical in both axes.
2. $2Y^3 = -X + 5$
   (not symmetrical about the Y-axis)
   $2(-Y)^3 = X + 5$
   $-2Y^3 = X + 5$
   (not symmetrical about the X-axis)
3. $Y = (-X)^4$
   $Y = X^4$ (symmetrical about the Y-axis)
   $-Y = X^4$ (not symmetrical about the X-axis)
4. $9X^2 + 4Y^2 = 35$
   $9(-X)^2 + 4(-Y)^2 = 35$
   $9X^2 + 4Y^2 = 35$
   (symmetrical around the X- and Y-axis and the origin)
5. $3(-Y)^2 = (-X) + 2$
   $3Y^2 = -X + 2$
   (making X negative changes the value, while making Y negative does not; symmetrical around the X-axis, but not the Y-axis or the origin)
6. $XY = \sqrt{2}$
   $(-X)(-Y) = \sqrt{2}$
   $XY = \sqrt{2}$
   (changing X or Y would change the value, but changing both does not; symmetrical about the origin, but not the X-or Y-axes)
7. $Y = |X| - 3$
   $(-Y) = |(-X)| - 3$
   $-Y = |X| - 3$
   (changing Y changes the value, but changing X does not; symmetrical about the Y-axis only)

Honors Lesson 3

1. $\frac{X}{10} = \frac{5}{9}$
   $X = \frac{50}{9} = 5.56$
   round to 6 and add:
   59.46°
2. $\frac{X}{10} = \frac{2}{9}$
   $X = \frac{20}{9} = 2.2$
   round to 2 and add:
   59.72°
3. $\frac{7}{10} = \frac{X}{9}$
   $X = \frac{63}{10} = 6.3$
   round to 6 and add: .8631

4. $\frac{3}{10} = \frac{X}{9}$

$X = \frac{27}{10} = 2.7$

round to 3 and add:

.8584

Honors Lesson 4

1. $4X^2 - 9Y^2 = (2X + 3Y)(2X - 3Y)$
2. $8X^3 - Y^3 = (2X - Y)(4X^2 + 2XY + Y^2)$
3. $(2X^3 + 2X^2) + (3X + 3) =$

   $2X^2(X + 1) + 3(X + 1) =$

   $(2X^2 + 3)(X + 1)$
4. $a^6 - b^3 = (a^2 - b)(a^4 + a^2b + b^2)$
5. $Y^4 - 8Y^2 + 12 =$

   $W^2 - 8W + 12 =$

   $(W - 6)(W - 2) =$

   $(Y^2 - 6)(Y^2 - 2)$
6. $64r^3 + 8w^3 = (4r + 2w)(16r^2 - 8rw + 4w^2)$

   $= 8(2r + w)(4r^2 - 2rw + w^2)$
7. $(B^3 + 2B^2) - (3B + 6) =$

   $B^2(B + 2) - 3(B + 2) =$

   $(B^2 - 3)(B + 2)$
8. $X + 5\sqrt{X} + 6 =$

   $W^2 + 5W + 6 =$

   $(W + 2)(W + 3) =$

   $(\sqrt{X} + 2)(\sqrt{X} + 3)$

Honors Lesson 5

You may have factored some of these in a different order, but your results should be the same.

1. $\frac{X^2Y^3}{X^2 - 4X - 5} \times \frac{2X^2 - 13X + 15}{XY^3} =$

   $\frac{\cancel{X}\cancel{X}\cancel{Y^3}}{(X + 1)\cancel{(X - 5)}} \times \frac{(2X - 3)\cancel{(X - 5)}}{\cancel{X}\cancel{Y^3}} =$

   $\frac{X}{(X + 1)} \times \frac{(2X - 3)}{1} = \frac{X(2X - 3)}{X + 1} = \frac{2X^2 - 3X}{X + 1}$

2. $\frac{4X^2 - 4}{X^2 + 4X - 5} \div \frac{X^3 + 2X^2}{3X + 15} =$

   $\frac{(2X + 2)(2X - 2)}{(X + 5)(X - 1)} \times \frac{3(X + 5)}{X^2(X + 2)} =$

   $\frac{4(X + 1)\cancel{(X - 1)}}{\cancel{(X + 5)}\cancel{(X - 1)}} \times \frac{3\cancel{(X + 5)}}{X^2(X + 2)} =$

   $\frac{12(X + 1)}{X^2(X + 2)}$

3. $\frac{X - 3}{-X^2 + 4X - 3} = -\frac{X - 3}{X^2 - 4X + 3} =$

   $-\frac{\cancel{X - 3}}{(X - 1)\cancel{(X - 3)}} = -\frac{1}{X - 1} = \frac{1}{1 - X}$

4. $\frac{2 - X}{X^3 - 8} = -\frac{\cancel{X - 2}}{\cancel{(X - 2)}(X^2 + 2X + 4)} =$

   $\frac{-1}{X^2 + 2X + 4}$

5. $\frac{a^{2X} - b^{2X}}{a^X + b^X} = \frac{\cancel{(a^X + b^X)}(a^X - b^X)}{\cancel{(a^X + b^X)}} = a^X - b^X$

6. $\frac{\cancel{X^{2n} + 1}}{1 - X^{4n}} \times \frac{1 - X^2}{\cancel{X^{2n} + 1}} \times \frac{X^{4n} - 1}{X + 1} =$

   $\frac{1}{1 - X^{4n}} \times \frac{1 - X^2}{1} \times \frac{X^{4n} - 1}{X + 1} =$

   $\frac{-1}{\cancel{X^{4n} - 1}} \times \frac{X^2 - 1}{-1} \times \frac{\cancel{X^{4n} - 1}}{X + 1}$

   $\frac{-1}{1} \times \frac{\cancel{(X + 1)}(X - 1)}{-1} \times \frac{1}{\cancel{X + 1}} =$

   $\frac{\cancel{-1}}{1} \times \frac{X - 1}{\cancel{-1}} \times \frac{1}{1} = X - 1$

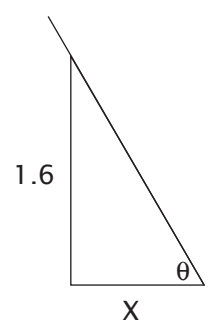
Honors Lesson 6

$\tan 60° = \frac{1.6}{X}$

1. $1.7321 = \frac{1.6}{X}$

   $1.7321X = 1.6$

   $X = .9237$ m



2. $\tan\theta = \frac{1.6}{1} = 1.6$

   $\theta \approx 58°$

3. $\tan\theta = \frac{16}{20}$

   $\tan\theta = .8000$

   $\theta = 38.7°$

4. done

5. $\tan 32° = \frac{X}{Y}$ $\quad$ $\tan 21° = \frac{X}{Y+50}$

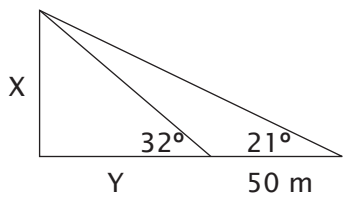
   $.6249 = \frac{X}{Y}$ $\quad$ $.3839 = \frac{.6249Y}{Y+50}$

   $X = .6249Y$ $\quad$ $(Y+50)(.3839) = .6249Y$

   $.3839Y + 19.195 = .6249Y$

   $19.195 = .241Y$

   $Y = 79.65$ meters



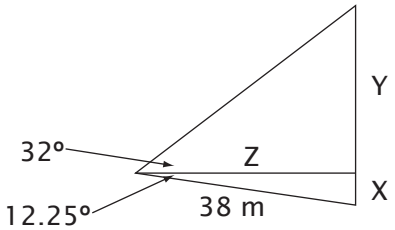
6. $12°15' = 12.25°$

$\sin 12.25° = \frac{X}{38}$ $\quad$ $\tan 12.25° = \frac{8.06}{Z}$ $\quad$ $\tan 32° = \frac{Y}{37.13}$

$.2122 = \frac{X}{38}$ $\quad$ $.2171 = \frac{8.06}{Z}$ $\quad$ $.6249 = \frac{Y}{37.13}$

$X = 8.06$ m $\quad$ $.2171Z = 8.06$ $\quad$ $Y = 23.20$ m

$Z = 37.13$ m



$Y + X = 23.20 + 8.06 = 31.26$ m

Honors Lesson 7

1. The angle of elevation from the ponds is the same as the angle of depression from the top of the mountain. (alternate interior angles)

$\tan 24° = \frac{350}{Y}$ $\quad$ $\tan 35° = \frac{350}{X}$

$.4452 = \frac{350}{Y}$ $\quad$ $.7002 = \frac{350}{X}$

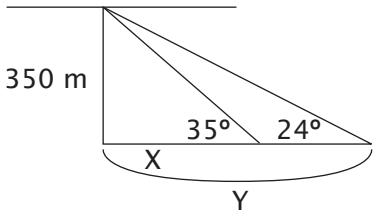
$.4452Y = 350$ $\quad$ $.7002X = 350$

$Y = 786.2$ m $\quad$ $X = 499.9$ m

$Y - X = 786.2 - 499.9 = 286.3$ m

There is more than one way to draw and solve many of these problems. Depending on the level of accuracy needed, you may also chose to round later in your calculations.

You can review significant digits in Algebra 1or Honors for Algebra 2 .



2. $42°30' = 42.5°$

   $36°45' = 36.75°$

   Adding an extra line 22 m above the ground is helpful here. It creates a rectangle with height 22 length X.

$\tan 36.75 = \frac{Y}{X}$ $\quad$ $\tan 42.5 = \frac{Y+22}{X}$

$.7467 = \frac{Y}{X}$ $\quad$ $.9163 = \frac{Y+22}{X}$

$X = \frac{Y}{.7467}$ $\quad$ $.9163X = Y + 22$

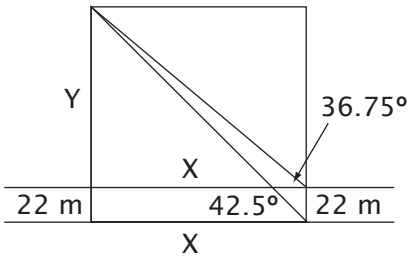
$.9163(\frac{Y}{.7467}) = Y + 22$

$1.2271Y = Y + 22$

$.2271Y = 22$

$Y = 96.9$ m

$Y + 22 = 96.9 + 22 = 118.9$ m above the ground



3. $\tan 36.75 = \frac{96.9}{X}$

$.7467 = \frac{96.9}{X}$

$X = \frac{96.9}{.7467}$

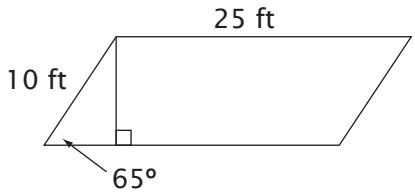
$X = 129.8$ m

4. $\sin 65° = \frac{X}{10}$

$.9063 = \frac{X}{10}$

$9.063 = X$

$A = 9.063 \times 25 = 226.58$ sq ft



5. $\tan 45° = \frac{Y}{X}$

$1 = \frac{Y}{X}$

$\tan 27° = \frac{Y}{85 + X}$

$.5095 = \frac{Y}{85 + X}$

$.5095 = \frac{Y}{85 + Y}$ $(X = Y)$

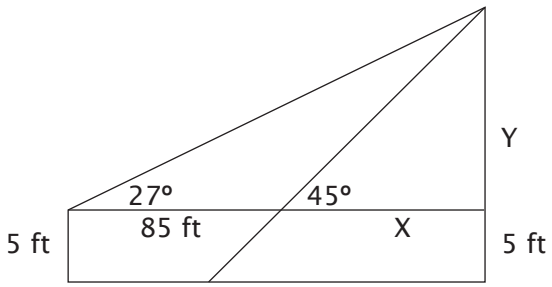
$.5095(85 + Y) = Y$

$43.31 + .5095Y = Y$

$43.31 = .4905Y$

$Y = 88.3$ ft

height $= 88.3 + 5 = 93.3$ ft

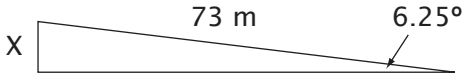


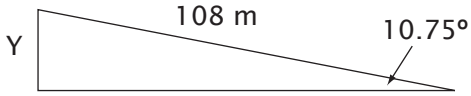
6. $\sin 6.25° = \frac{X}{73}$ $\sin 10.75° = \frac{Y}{108}$

$.1089 = \frac{X}{73}$ $.1865 = \frac{Y}{108}$

$X = 7.95$ m $Y = 20.14$

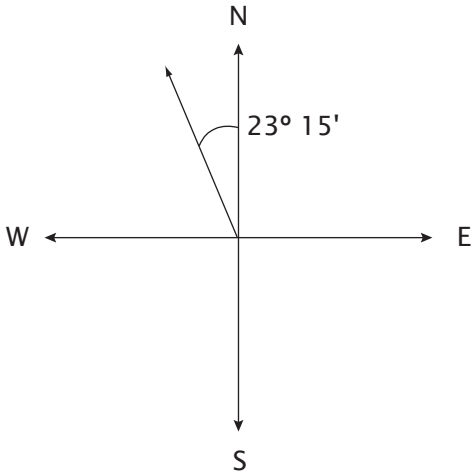
$7.95 + 20.14 = 28.09$ m



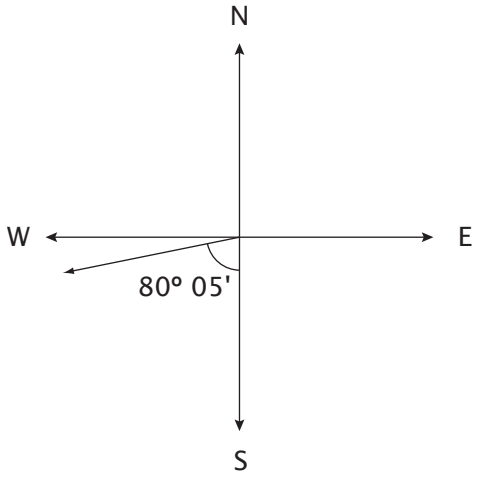


Honors Lesson 8

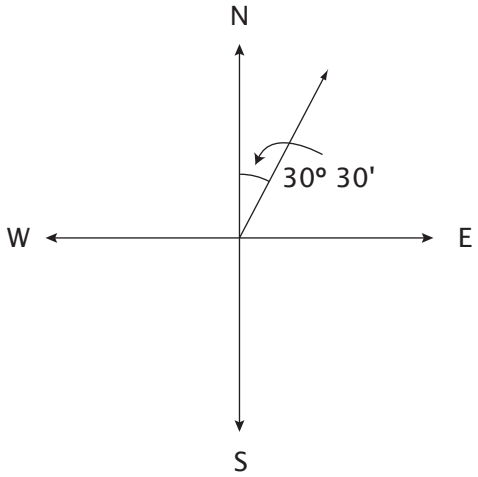
1.



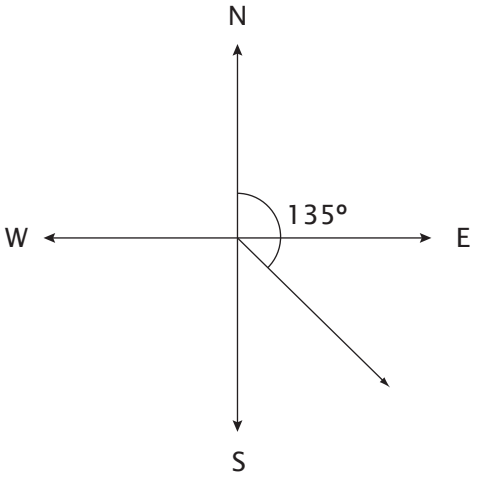
2.



3.



4.



Honors Lesson 9

1. $\sin 25.17° = \frac{X}{65}$ $\quad$ $\tan 25.17° = \frac{27.64}{Y}$

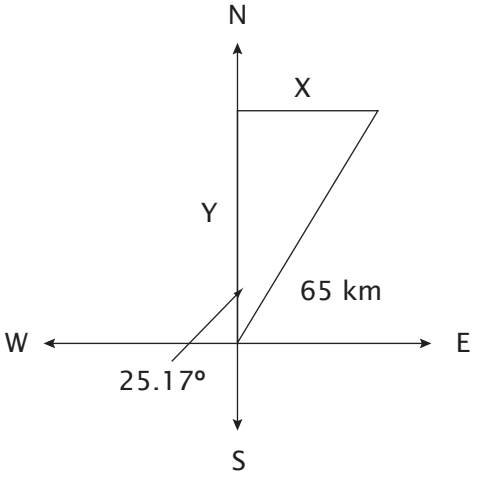
$.4253 = \frac{X}{65}$ $\quad$ $.4699 = \frac{27.64}{Y}$

$X = 27.64$ km $\quad$ $.4699Y = 27.64$

$Y = 58.82$ km

trip back = $27.64 + 58.82 = 84.46$ km

round trip = $84.46 + 65 = 151.46$ km



2. The turn between a heading of 120° and 30° is a 90° angle, so we can use the Pythagorean theorem:

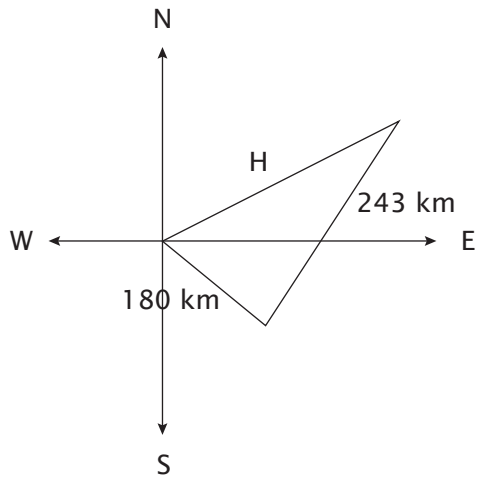
$180^2 + 243^2 = H^2$

$32{,}400 + 59{,}049 = H^2$

$91{,}449 = H^2$

$H \approx 302$ km

Total trip $= 180 + 243 + 302 = 725$ km



3. $\sin 28.83° = \frac{Y}{95}$ $\quad$ $\cos 28.83° = \frac{X}{95}$

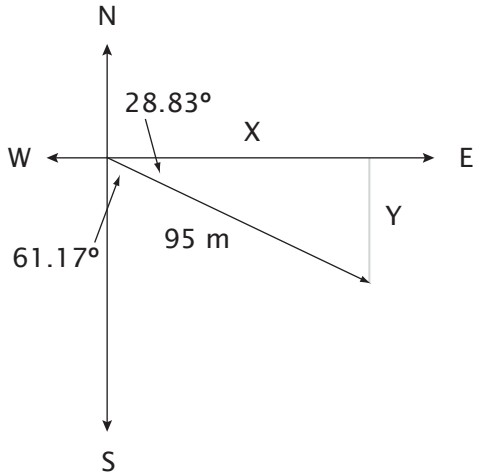
$.4822 = \frac{Y}{95}$ $\quad$ $.8761 = \frac{X}{95}$

$Y = 45.81$ m $\quad$ $.8049X = 59.57$

$X = 83.23$ m

(The Pythagorean theorem may be used instead to find the third side of the triangle.)

$A = \frac{1}{2}(45.81)(83.23) = 1{,}906$ sq m



4. $\tan 14.83° = \frac{Y}{60.3}$ $\quad$ $60.3^2 + 15.97^2 = H^2$

$.2648 = \frac{Y}{60.3}$ $\quad$ $3{,}891.1309 = H^2$

$Y = 15.97$ $\quad$ $H \approx 62.38$

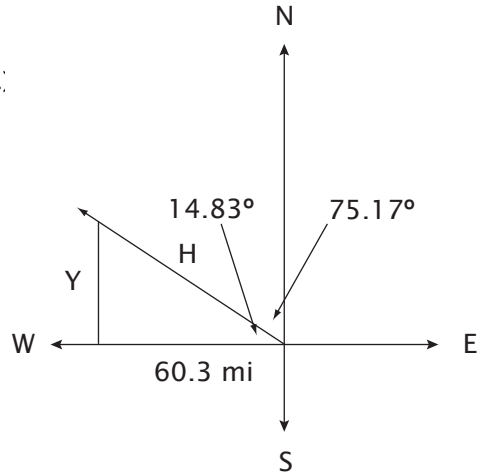
car went $60.3 + 15.97 = 76.27$ mi

car time $= \frac{76.27}{55} \approx 1.39$ hours

bus time $= \frac{62.38}{50} \approx 1.25$ hours

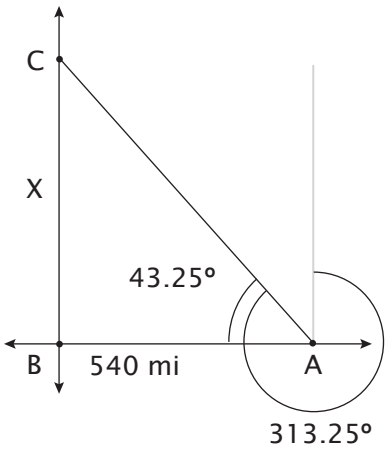
$1.39 - 1.25 = .14$ hrs, or 8.4 minutes

There may be more than one way to draw these problems.



Honors Lesson 10

1. $180 \times 3 = 540$ mi
   $313.25 - 270 = 43.25°$
   $\tan 43.25 = \frac{X}{540}$
   $.9407 = \frac{X}{540}$
   $X = 507.98$ mi



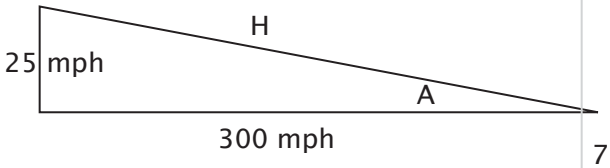
2.

$300^2 + 25^2 = H^2$
$90{,}000 + 625 = H^2$
$90{,}625 = H^2$
$H = 301.04$ mph

$\tan A = \frac{25}{300}$
$A = 4.764° = 4°46'$

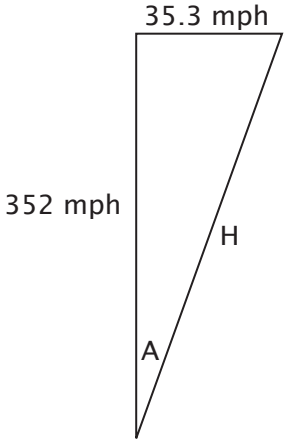
$270° - 4.76 = 265.24$

Heading is 265°14'



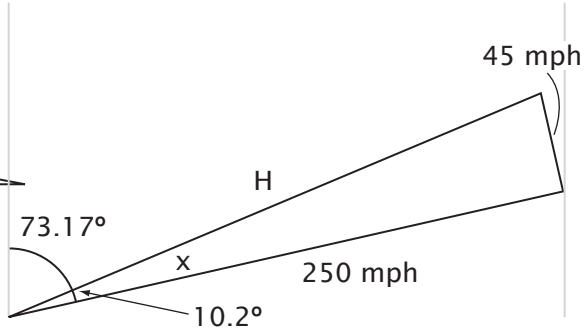
3. $352^2 + 35.3^2 = H^2$
   $123{,}904 + 1{,}246.09 = H^2$
   $125{,}150.09 = H^2$
   $H \approx 353.77$ mph

   $\tan A = \frac{35.3}{352}$
   $A = 5.7267° \approx 5°44'$



4. $\tan X = \frac{45}{250} = .18$
   $X = 10.2°$ or $10°12'$
   $250^2 + 45^2 = H^2$
   $H \approx 254$ mph

The plane is blown 10°12' toward the the west, so its new direction of travel will be:
$73°10' - 10°12' = 62°58'$



5. $\tan X = \frac{6}{27} = .2222$

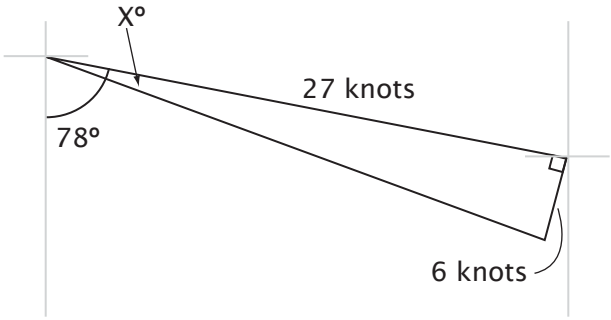
   $X = 12.5°$

   $27^2 + 6^2 = H^2$

   $H = 27.7$

   $78° - 12.5° = 65.5°$

   Direction is S 65.5° E at 27.7 knots



8. $2 - 2\cos A\cos B - 2\sin A\sin B = 2 - 2\cos(A - B)$

   $-2\cos A\cos B - 2\sin A\sin B = -2\cos(A - B)$

   $\cos A\cos B + \sin A\sin B = \cos(A - B)$

Challenge question:

The same drawings can be used, but start with two smaller angles and add up to the larger one. You should end up with this:

$\cos A\cos B - \sin A\sin B = \cos(A + B)$

Honors Lesson 11

1. done
2. done
3. $(\cos B, \sin B)$
4. $(\cos B, \sin B)$
5. $\overline{CD}^2 = (X_1 - X_2)^2 + (Y_1 - Y_2)^2$

   $= (\cos B - \cos A)^2 + (\sin B - \sin A)^2$

   $= \cos^2 B - 2\cos B\cos A + \cos^2 A + \sin^2 B - 2\sin B\sin A + \sin^2 A$

   (Combine $\sin^2 B + \cos^2 B$ to make 1 and combine $\sin^2 A + \cos^2 A$ to make 1.)

   $= 2 - 2\cos B\cos A - 2\sin B\sin A$
6. done
7. $\overline{FE}^2 = (Y_1 - Y_2)^2 + (X_1 - X_2)^2$

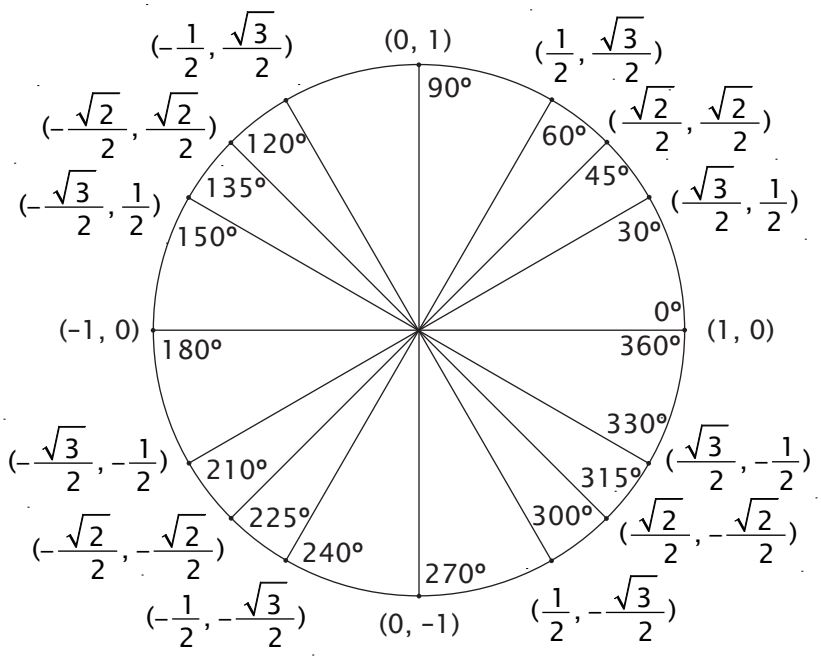
   $= (\sin(A - B) - 0)^2 + (\cos(A - B) - 1)^2$

   $= 1 + \sin^2(A - B) + \cos^2(A - B) - 2\cos(A - B)$

   (Combine $\sin^2(A - B) + \cos^2(A - B)$ to make 1.)

   $= 2 - 2\cos(A - B)$

Honors Lesson 12



1. all
2. sin, csc
3. tan, cot
4. cos, sec

Honors Lesson 13

1. $(\frac{1}{2})(15)(16.4)\sin 52° = 96.9\ \text{cm}^2$
2. $(\frac{1}{2})(12.5)(8.3)\sin 69° = 48.4\text{m}^2$
3. $(\frac{1}{2})(28)(31)\sin 43° = 296.0\ \text{cm}^2$
4. $180° - (60° + 60°) = 60°$

   $(\frac{1}{2})(7)(7)\sin 60° = 21.2\ \text{mm}^2$

Honors Lesson 14

1. $X^3 - 2X^2 - X + 2$

   $P = 2 \rightarrow \{\pm 1, \pm 2\}$ poss. roots $\{\pm 1, \pm 2\}$

   $q = 1 \rightarrow \{\pm 1\}$

   try $X = 1$:

   $(1)^3 - 2(1)^2 - (1) + 2 = 0$

   $1 - 2 - 1 + 2 = 0$

   $0 = 0$ yes

   1 is a root, so $(X-1)$ must be a factor.

You may use different roots for the initial tries, but final answers should be the same.

$$
\begin{array}{r|rrrr}
 & X^2 & -X & -2 & \\
\hline
X-1 & X^3 & -2X^2 & -X & +2 \\
 & X^3 & -X^2 & & \\
\hline
 & & -X^2 & -X & \\
 & & -X^2 & +X & \\
\hline
 & & & -2X & +2 \\
 & & & -2X & +2 \\
\hline
 & & & & 0
\end{array}
$$

$X^2 - X - 2 = (X+1)(X-2)$

roots are 1, −1 and 2

2. $X^3 + 3X^2 - X - 3$

   $p = -3 \rightarrow \{\pm 1, \pm 3\}$ poss. roots $\{\pm 1, \pm 3\}$

   $q = 1 \rightarrow \{\pm 1\}$

   try $X = -3$:

   $(-3)^3 + 3(-3)^2 - (-3) - 3 = 0$

   $-27 + 27 + 3 - 3 = 0$

   $0 = 0$ yes

   −3 is a root, so $(X+3)$ must be a factor.

$$
\begin{array}{r|rrrr}
 & X^2 & & -1 & \\
\hline
X+3 & X^3 & +3X^2 & -X & -3 \\
 & X^3 & +3X^2 & & \\
\hline
 & & 0 & -X & -3 \\
 & & & -X & -3 \\
\hline
\end{array}
$$

$X^2 - 1 = (X+1)(X-1)$

roots are 1, −1 and −3

3. $X^4 - 5X^2 + 4$

   $p = 4 \rightarrow \{\pm 1, \pm 2, \pm 4\}$ poss. roots $\{\pm 1, \pm 2, \pm 4\}$

   $q = 1 \rightarrow \{\pm 1\}$

   try $X = 1$:

   $(1)^4 - 5(1)^2 + 4 = 0$

   $1 - 5 + 4 = 0$

   $0 = 0$ yes

   1 is a root, so $(X-1)$ must be a factor.

$$
\begin{array}{r|rrrrr}
 & X^3 & +X^2 & -4X & -4 & \\
\hline
X-1 & X^4 & +0X^3 & -5X^2 & +0X & +4 \\
 & X^4 & -X^3 & & & \\
\hline
 & & X^3 & -5X^2 & & \\
 & & X^3 & -X^2 & & \\
\hline
 & & & -4X^2 & +0X & \\
 & & & -4X^2 & +4X & \\
\hline
 & & & & -4X & +4 \\
 & & & & -4X & +4 \\
\hline
\end{array}
$$

Repeat with the results of the division:

$X^3 + X^2 - 4X - 4$

$p = 4 \rightarrow \{\pm 1, \pm 2 \pm 4\}$ poss. roots $\{\pm 1, \pm 2, \pm 4\}$

$q = 1 \rightarrow \{\pm 1\}$

try $X = 2$:

$(2)^3 + (2)^2 - 4(2) - 4 = 0$

$8 + 4 - 8 - 4 = 0$

$0 = 0$ yes

2 is a root, so $(X-2)$ must be a factor.

$$
\begin{array}{r|rrrr}
 & X^2 & +3X & +2 & \\
\hline
X-2 & X^3 & +X^2 & -4X & -4 \\
 & X^3 & -2X^2 & & \\
\hline
 & & 3X^2 & -4X & \\
 & & 3X^2 & -6X & \\
\hline
 & & & 2X & -4 \\
 & & & 2X & -4 \\
\hline
\end{array}
$$

$X^2 + 3X + 2 = (X+1)(X+2)$

all roots are 1, 2, −1 and −2

4. $3X^3 + 6X^2 - 15X - 18$

$p = -18 \rightarrow \{\pm1, \pm2, \pm3, \pm6, \pm9, \pm18\}$

$q = 3 \rightarrow \{\pm1, \pm3\}$

poss. roots:

$\{\pm1, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm2, \pm3, \pm6, \pm9, \pm18\}$

try $X = 1$:

$3(1)^3 + 6(1)^2 - 15(1) - 18 = 0$

$3 + 6 - 15 - 18 = 0$

$-24 = 0$ no

try $X = 2$:

$3(2)^3 + 6(2)^2 - 15(2) - 18 = 0$

$24 + 24 - 30 - 18 = 0$

$0 = 0$ yes

2 is a root, so $(X - 2)$ must be a factor

$$
\begin{array}{r|rrrr}
 & 3X^2 & +12X & +9 & \\
\hline
X-2 & 3X^3 & +6X^2 & -15X & -18 \\
 & \underline{3X^2} & \underline{-6X^2} & & \\
 & & 12X^2 & -15X & \\
 & & \underline{12X^2} & \underline{-24X} & \\
 & & & 9X & -18 \\
 & & & \underline{9X} & \underline{-18}
\end{array}
$$

$3X^2 + 12X + 9 = 3(X^2 + 4X + 3) = 3(X + 3)(X + 1)$

roots are $2, -3, -1$

Honors Lesson 15

1.
$$
\begin{array}{r|rrr}
2 & 3 & -4 & 6 \\
 & & 6 & 4 \\
\hline
 & 3 & 2 & 10
\end{array}
$$

$3X + 2$ r 10

2.
$$
\begin{array}{r|rrr}
1 & 3 & -2 & 1 \\
 & & 3 & 1 \\
\hline
 & 3 & 1 & 2
\end{array}
$$

$3X + 1$ r 2

3.
$$
\begin{array}{r|rrr}
-2 & 3 & -4 & 6 \\
 & & -6 & 20 \\
\hline
 & 3 & -10 & 26
\end{array}
$$

$3X - 10$ r 26

4.
$$
\begin{array}{r|rrr}
-3 & 2 & -7 & 2 \\
 & & -6 & 39 \\
\hline
 & 2 & -13 & 41
\end{array}
$$

$2X - 13$ r 41

5.
$$
\begin{array}{r|rrrrr}
-3 & 1 & -3 & 2 & -3 & 1 \\
 & & -3 & 18 & -60 & 189 \\
\hline
 & 1 & -6 & 20 & -63 & 190
\end{array}
$$

$X^3 + -6X^2 + 20X - 63$ r 190

6.
$$
\begin{array}{r|rrrrr}
-2 & 3 & 0 & -1 & 0 & 0 \\
 & & -6 & 12 & -22 & 44 \\
\hline
 & 3 & -6 & 11 & -22 & 44
\end{array}
$$

$3X^3 - 6X^2 + 11X - 22$ r 44

Honors Lesson 16

1. -1 has multiplicity of 1

   -2 has multiplicity of 1

   -3 has multiplicity of 4

2. $X^4 - X^2 =$

   $(X^2 + X)(X^2 - X) =$

   $(X)(X + 1)(X)(X - 1)$

   roots:

   0 has a multiplicity of 2

   -1 has a multiplicity of 1

   1 has a multiplicity of 1

3. roots :

i	has a multiplicity of 2
$-i$	has a multiplicity of 1
-4	has a multiplicity of 6
-2	has a multiplicity of 1

4. multiply out the four factors indicated by the given roots:

$(X+i)(X-i)(X+2)(X-2) =$

$X^4 - 3X^2 - 4$

Honors Lesson 17

1.
$$\begin{array}{r|rrrr} 1 & 6 & 5 & -3 & 2 \\ & & 6 & 11 & 8 \\ \hline & 6 & 11 & 8 & 10 \end{array}$$

$6(1)^3 + 5(1)^2 - 3(1) + 2 =$

$6 + 5 - 3 + 2 = 10$

2.
$$\begin{array}{r|rrrr} 2 & 1 & 4 & 1 & -6 \\ & & 2 & 12 & 26 \\ \hline & 1 & 6 & 13 & 20 \end{array}$$

$(2)^3 + 4(2)^2 + (2) - 6 =$

$8 + 16 + 2 - 6 = 20$

3.
$$\begin{array}{r|rrrr} -3 & 1 & 4 & 1 & -6 \\ & & -3 & -3 & 6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$(-3)^3 + 4(-3)^2 + (-3) - 6 =$

$-27 + 36 - 3 - 6 = 0$

4.
$$\begin{array}{r|rrrr} -7 & 2 & 13 & -8 & -7 \\ & & -14 & 7 & 7 \\ \hline & 2 & -1 & -1 & 0 \end{array} \rightarrow \text{yes}$$

$Y = 0$

5.
$$\begin{array}{r|rrrr} 1 & 2 & 13 & -8 & -7 \\ & & 2 & 15 & 7 \\ \hline & 2 & 15 & 7 & 0 \end{array} \rightarrow \text{yes}$$

$Y = 0$

6.
$$\begin{array}{r|rrrr} -1 & 2 & 13 & -8 & -7 \\ & & -2 & -11 & 19 \\ \hline & 2 & 11 & -19 & 12 \end{array} \rightarrow \text{no}$$

$Y = 12$

Honors Lesson 18

1. Airplane :

$X = r\cos\theta$

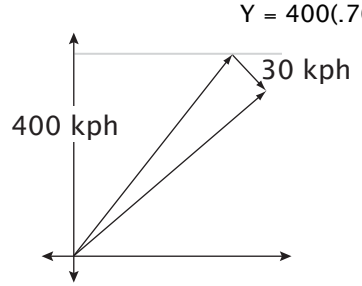
$X = 400\cos(90° - 40°)$

$X = 400(.6428) = 257.1$

$Y = r\ \sin\theta$

$Y = 400 \sin 50°$

$Y = 400(.7660) = 306.4$



Wind :

$X = r\cos\theta$

$X = 30\cos(-10)$

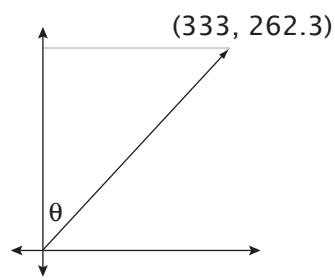
$X = 30(.9848) = 29.5$

$Y = r\ \sin\theta$

$Y = 30\sin(-10)$

$Y = 30(-.1736) = -5.2$

$$\begin{array}{l} 257.1X + 306.4Y \\ \underline{+\ 29.5X - \ \ 5.2Y} \\ 286.6X + 301.2Y \end{array}$$



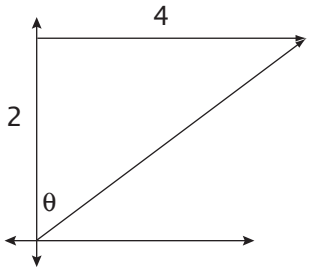
$\tan\theta = \dfrac{301.2}{286.6}$ $\qquad r^2 = 286.6^2 + 301.2^2$

$\tan\theta = 1.0509$ $\qquad r^2 = 172{,}861$

$\theta = 46.4°$ $\qquad r \approx 415.8$ kph

$(415.8 \text{ kph, N } 43.6° \text{ E})$

2. $$\begin{array}{r} 0X + 2Y \\ +\ 4X + 0Y \\ \hline 4X + 2Y \end{array}$$



$\tan\theta = \frac{4}{2} = 2$ $\qquad \frac{X}{75} = \tan 63.43$

$\theta = 63.43°$ $\qquad \frac{X}{75} = 2$

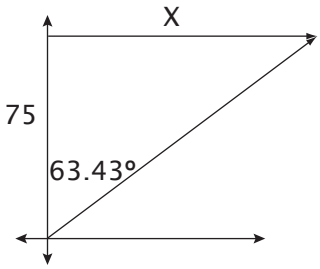
$X = 150$

$75^2 + 150^2 = r^2$

$28{,}125 = r^2$

$167.71 = r$

(63.4°, 167.7)

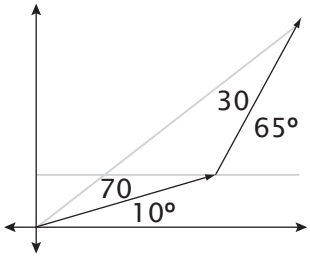


3. Bob : $X = 70\cos 10° = 68.94$

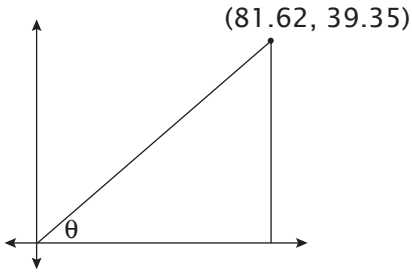
$Y = 70\sin 10° = 12.16$

Steve : $X = 30\cos 65° = 12.68$

$Y = 30\sin 65° = 27.19$



$$\begin{array}{rcl} 68.94X & + & 12.16Y \\ +\ 12.68X & + & 27.19Y \\ \hline 81.62X & + & 39.35Y \end{array}$$



$r^2 = 81.62^2 + 39.35^2$

$r^2 = 8{,}210.24$

$r = 90.6$

$\tan\theta = \frac{39.35}{81.62} = .4821$

$\theta = 25.74°$

(90.6, 25.74°)

Rounds to:

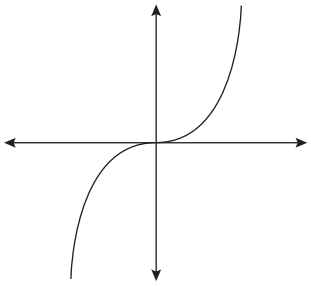
(91, 26°)

Honors Lesson 19

(You may have used other values for the charts.)

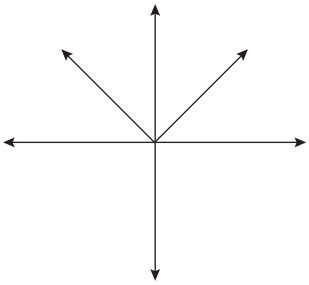
1. $f(X) = X^3$

X	X^3
−3	−27
−2	−8
−1	−1
0	0
1	1
2	8
3	27

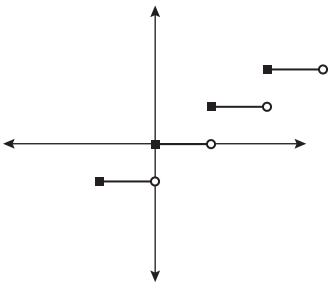


2. $f(X) = |X|$

X	| X |
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3

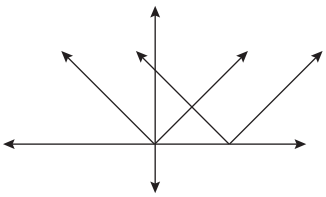


3. $X = 2$; $f(X) = 2$
4. $X = 2.4$; $f(X) = 2$
5. $X = 2.8$; $f(X) = 2$
6. $1 \le X < 2$; $f(X) = 1$
7. $0 \le X < 1$; $f(X) = 0$
8. $-1 \le X < 0$; $f(X) = -1$

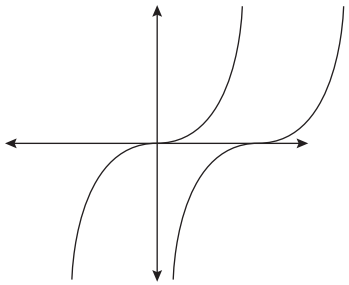


Honors Lesson 20

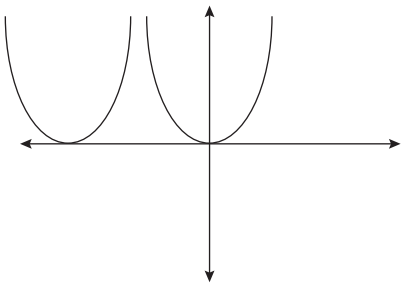
1. done
2. translated right three places



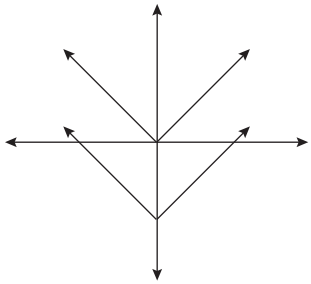
3. translated right two places



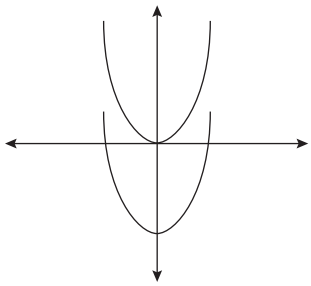
4. translated left four places



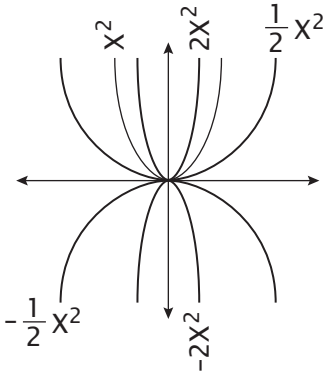
5. done
6. translated down two places



7. translated down three places



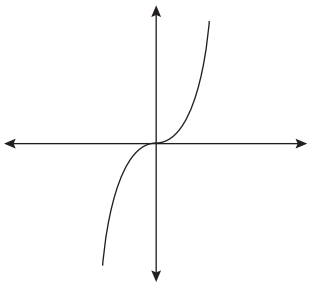
8-12.



Honors Lesson 21

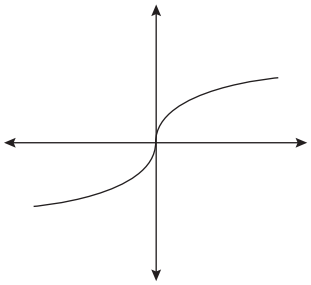
1. yes
2. no
3. yes
4. no
5. no
6. $f(X) = X^3$

   $Y = X^3$



   $X = \sqrt[3]{Y}$

   inverse : $Y = \sqrt[3]{X}$; function

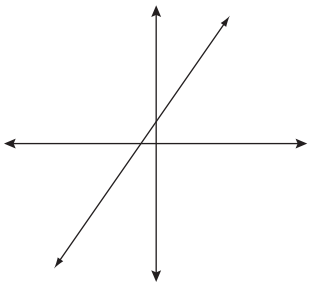


7. $f(X) = 2X + 1$

   $Y = 2X + 1$

   $Y - 1 = 2X$

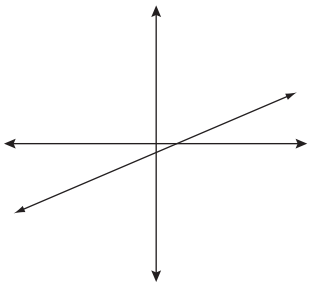
   $\frac{Y-1}{2} = X$



   inverse : $Y = \frac{X-1}{2}$; function

   $Y = \frac{X}{2} - \frac{1}{2}$

   $Y = \frac{1}{2}X - \frac{1}{2}$



Honors Lesson 22

1. $f(3) = e^3 = 20.086$

   $f(3) = \ln(3) = 1.099$

   $\log_e(3) = 1.099$ and $e^{1.099} \approx 3$

2. $f(4) = e^4 = 54.598$

   $f(4) = \ln(4) = 1.386$

   $\log_e(4) = 1.386$ and $e^{1.386} \approx 4$

Honors Lesson 23

1. $f(X) = \frac{X+7}{X-4}$ continuous when $X \neq 4$

2. $f(X) = \frac{5X-1}{X^2-3X-2}$

   First, find values of X that will make the denominator equal to zero:

   $X^2 - 3X - 2 = 0$

   Solve the equation using the quadratic formula:

   $X = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$

   $= \frac{3 \pm \sqrt{9-(-8)}}{2}$

   $= \frac{3 \pm \sqrt{17}}{2}$

   Continuous for all X except $\frac{3+\sqrt{17}}{2}$ and $\frac{3-\sqrt{17}}{2}$.

3. $f(X) = \frac{6X+1}{\sqrt{X-5}}$

   Continuous where $X > 5$.
   If $X = 5$, the denominator is 0, if $X < 5$, the denominator is imaginary.

4. $f(X) = \frac{\sqrt{3+X}}{X+8}$

   $X \geq -3$; otherwise the numerator is imaginary.
   $X \neq -8$, because -8 makes the denominator 0.
   The first restriction includes the second.

Honors Lesson 24

1. $h = 3\sin\frac{\pi}{6}(0) + 2$

   $h = 3\sin 0 + 2$

   $h = 3(0) + 2$

   $h = 0 + 2$

   $h = 2$

2. $h = 3\sin\frac{\pi}{6}(1) + 2$

   $h = 3\sin\frac{\pi}{6} + 2$

   $h = 3(.5) + 2$

   $h = 1.5 + 2$

   $h = 3.5$

3. $h = 3\sin\frac{\pi}{6}(3) + 2$

   $h = 3(1) + 2$

   $h = 3 + 2$

   $h = 5$

4. $h = 3\sin\frac{\pi}{6}(6) + 2$

   $h = 3(0) + 2$

   $h = 2$

5. $h = 3\sin\frac{\pi}{6}(8) + 2$

   $h = 3(-.8660) + 2$

   $h = -2.598 + 2$

   $h = -.598$

6. $h = 3\sin\frac{\pi}{6}(9) + 2$

   $h = 3(-1) + 2$

   $h = -3 + 2$

   $h = -1$

7. $h = 3\sin\frac{\pi}{6}(12) + 2$

   $h = 3(0) + 2$

   $h = 0 + 2$

   $h = 2$

8. $h = 3\sin\frac{\pi}{6}(15) + 2$

   $h = 3(1) + 2$

   $h = 3 + 2$

   $h = 5$

9. 3

   This means that the highest point of the graph is three units above a line representing the midpoint of the graph. The lowest point of the graph is three units below the line. So, there are six meters between high and low tide.

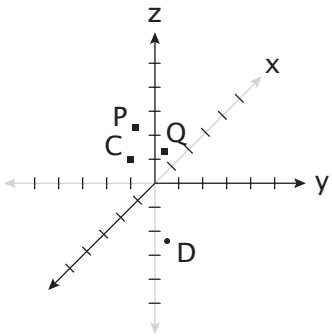
10. 12
There are 12 hours from one high tide to the next.

Honors Lesson 25

1. $f(X) = 2X^2$
parabola :
horizontally widens or narrows the parabola
2. $f(X) = X^2 + 1$
parabola :
translates the graph up or down
3. $f(X) = \frac{6}{X}$
hyperbola :
horizontally widens or narrows the hyperbola
4. $f(X) = \sin X + 2$
sine curve:
translates the graph up or down
5. $f(X) = 3\sin X$
sine curve:
vertically stretches or shrinks the curve
6. $f(X) = \sin 2X$
sine curve:
horizontally widens or narrows the curve

Honors Lesson 26

1. on graph



2. (0, −1, 1)(2, 2, −1)
$d = \sqrt{(2-0)^2 + (2--1)^2 + (-1-1)^2}$
$= \sqrt{4+9+4}$
$= \sqrt{17}$
≈ 4.12
3. midpoint $= (\frac{0+2}{2}, \frac{-1+2}{2}, \frac{1+-1}{2})$
$= (\frac{2}{2}, \frac{1}{2}, \frac{0}{2})$
$= (1, \frac{1}{2}, 0)$
4. on graph
5. (1, 0, 3)(−2,−1, 0)
$d = \sqrt{(-2-1)^2 + (-1-0)^2 + (0-3)^2}$
$= \sqrt{9+1+9}$
$= \sqrt{19}$
≈ 4.36
6. midpoint $= (\frac{1-2}{2}, \frac{0-1}{2}, \frac{3+0}{2})$
$= (\frac{-1}{2}, \frac{-1}{2}, 1\frac{1}{2})$
7. $(X-4)^2 + (Y+2)^2 + (Z-1)^2 = 2^2$
$X^2 - 8^X + 16 + Y^2 + 4Y + 4 + Z^2 - 2Z + 1 = 4$
$X^2 + Y^2 + Z^2 - 8X + 4Y - 2Z + 21 = 0$
8. $X^2 + Y^2 + Z^2 - 2X + 4Y + 2Z + 2 = 0$
Group each variable together, and put the constant on the right:
$X^2 - 2X + Y^2 + 4Y + Z^2 + 2Z = -2$
Complete the square in each variable:
$X^2 - 2X + 1 + Y^2 + 4Y + 4 + Z^2 + 2Z + 1 = -2 + 1 + 4 + 1$
$(X-1)^2 + (Y+2)^2 + (Z+1)^2 = 4$
center : (1,−2,−1)
radius : 2

Honors Lesson 27

1. $((1)-1)-\frac{1}{2}((1)-1)^2+\frac{1}{3}((1)-1)^3 =$

$$0-\frac{1}{2}(0)+\frac{1}{3}(0) =$$
$$0+0+0=0$$
$$\ln 1 = 0$$

2. $2\left(\frac{2-1}{2+1}+\frac{1}{3}\left(\frac{2-1}{2+1}\right)^3+\frac{1}{5}\left(\frac{2-1}{2+1}\right)^5\right) =$

$$2\left(\frac{1}{3}+\frac{1}{3}\left(\frac{1}{3}\right)^3+\frac{1}{5}\left(\frac{1}{3}\right)^5\right) =$$
$$2\left(\frac{1}{3}+\left(\frac{1}{3}\right)\left(\frac{1}{27}\right)+\left(\frac{1}{5}\right)\left(\frac{1}{243}\right)\right) =$$
$$2\left(\frac{1}{3}+\frac{1}{81}+\frac{1}{1215}\right) = .6930$$
$$\ln 2 \approx .6931$$

3. $\left(1-\frac{1}{2}\right)+\frac{1}{2}\left(1-\frac{1}{2}\right)^2+\frac{1}{3}\left(1-\frac{1}{2}\right)^3 =$

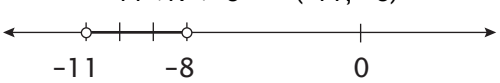
$$\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1}{3}\left(\frac{1}{2}\right)^3 =$$
$$\frac{1}{2}+\frac{1}{8}+\frac{1}{24} = .6667$$
$$\ln\frac{1}{\frac{1}{2}} = \ln 2 \approx .6931$$

Honors Lesson 28

1. $-4 < \frac{X-1}{3} < -3$

$-12 < X-1 < -9$

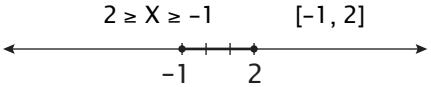
$-11 < X < -8$ $\quad (-11, -8)$



2. $1 \le \frac{X-5}{-3} \le 2$

$-3 \ge X-5 \ge -6$

$2 \ge X \ge -1$ $\quad [-1, 2]$



3. $2 \le \frac{4-X}{-2} \le 6$

$-4 \ge 4-X \ge -12$

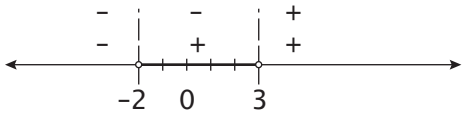
$-8 \ge -X \ge -16$

$8 \le X \le 16$ $\quad [8, 16]$

4. $X^2-X-6<0$

$(X-3)(X+2)<0$

We are looking for values of X that yield negative products, so the factors must come from the area where the signs are different, so $-2 < X < 3$ or $(-2, 3)$



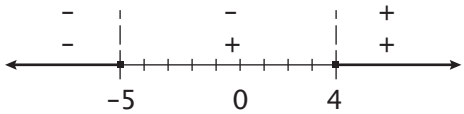
5. $X^2+X-20 \ge 0$

$(X+5)(X-4) \ge 0$

Signs must be the same:

$X \le -5$ and $X \ge 4$

$(-\infty, -5] \cup [4, \infty)$



Honors Lesson 29

1. try −2: $\quad$ try 0:

$\frac{3X+2}{X+1} \le 1$ $\quad \frac{3(0)+2}{0+1} \le 1$

$\frac{3(-2)+2}{(-2)+1} \le 1$ $\quad \frac{2}{1} \le 1$

$\frac{-6+2}{-1} \le 1$ $\quad 2 \le 1 \rightarrow$ no

$\frac{-4}{-1} \le 1$

$4 \le 1 \rightarrow$ no

2. It signifies that the value of −1 is not included in the solution.

3. It signifies that the value of $-\frac{1}{2}$ is included in the solution.

4. $\frac{3X+2}{X+1} \le 1$

$3X+2 \le X+1$

$3X \le X-1$

$2X \le -1$

$X \le -\frac{1}{2}$

If $X < -1$, then $X + 1$ would be negative. In that case, multiplying by $X + 1$ would have reversed the direction of the inequality.

5. $\frac{2X-4}{X+2} < -3$

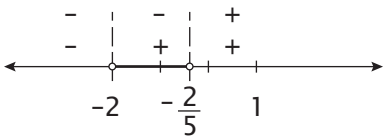
$\frac{2X-4}{X+2} + \frac{3X+6}{X+2} < 0$

$\frac{5X+2}{X+2} < 0$

$5X+2 < 0 \qquad X+2 < 0$

$5X < -2 \qquad X < -2$

$X < -\frac{2}{5}$



$(-2, -\frac{2}{5})$

6. $\frac{X+6}{X-1} > 4$

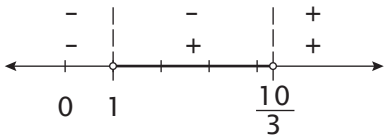
$\frac{X+6}{X-1} - \frac{4X-4}{X-1} > 0$

$\frac{-3X+10}{X-1} > 0$

$-3X+10 > 0 \qquad X-1 > 0$

$-3X > -10 \qquad X > 1$

$X < \frac{10}{3}$



$(1, \frac{10}{3})$

Honors Lesson 30

1. $f(A) = 2A^2 - A + 6$

$$f(A+h) = 2(A+h)^2 - (A+h) + 6$$
$$= 2(A^2 + 2Ah + h^2) - (A+h) + 6$$
$$= 2A^2 + 4Ah + 2h^2 - (A+h) + 6$$
$$f'(A) = \lim_{h\to 0} \frac{[2A^2 + 4Ah + 2h^2 - (A+h) + 6] - [2A^2 - A + 6]}{h}$$
$$= \lim_{h\to 0} \frac{4Ah + 2h^2 - h}{h}$$
$$= \lim_{h\to 0} 4A + 2h - 1$$
$$= 4A - 1$$

2. $f(X) = 3X^2 - 5X + 4$

$$f(X+h) = 3(X+h)^2 - 5(X+h) + 4$$
$$= 3(X^2 + 2Xh + h^2) - 5X - 5h + 4$$
$$= 3X^2 + 6Xh + 3h^2 - 5X - 5h + 4$$
$$f'(A) = \lim_{h\to 0} \frac{[3X^2 + 6Xh + 3h^2 - 5X - 5h + 4] - [3X^2 - 5X + 4]}{h}$$
$$= \lim_{h\to 0} \frac{6Xh + 3h^2 - 5h}{h}$$
$$= \lim_{h\to 0} 6X + 3h - 5$$
$$= 6X - 5$$

When you study calculus, you will find that there are shortcuts for problems such as these.