Zero Activities

X X^2

Use these activities if your student is working through *Pre-Algebra* or *Algebra 1* to teach and reinforce the concept of zero at a more advanced level.

Undefined: Division by Zero

Read the following mini-lesson with your student.

Division is the inverse, or opposite, of multiplication. Multiplication involves putting equal groups together, while division involves separating into equal groups. Because of this relationship, you can write equivalent equations, such as:

$$2 \cdot 6 = 12 \qquad 12 \div 2 = 6 \qquad 6 \cdot 2 = 12 \qquad 12 \div 6 = 2$$

These equations are related facts because they show the same relationship with the same three numbers.

Consider what happens when you write equivalent equations with zero. If you were given the equation $15 \div 3 = 5$, you could write two related multiplication facts: $3 \cdot 5 = 15$ and $5 \cdot 3 = 15$.

Suppose you were given the equation $0 \div 4 = 0$. You could rewrite this as $4 \cdot 0 = 0$ and $0 \cdot 4 = 0$. What would happen if you were asked to rewrite $2 \div 0$? This would be the same as saying that $0 \cdot ? = 2$ or $? \cdot 0 = 2$.

There is no number you can multiply by two that equals zero because any number multiplied by zero equals zero. Therefore, division by zero is not possible. It is considered to be *undefined*.

Next, have your student perform the following division problems on a calculator and write the response the calculator generates.

$\frac{-1302}{3}$ = -434

$\frac{154}{0}$ = Cannot Divide by Zero; Error; Undefined

$\frac{468}{-12}$ = -39

$\frac{685}{0}$ = Cannot Divide by Zero; Error; Undefined

$\frac{0}{27}$ = 0

Divison as a Repeated Subtraction

Have your student read the mini-lesson. Then answer each question completely.
This activity can also be completed as a discussion with your student.

Repeated subtraction is one way to model division with positive integers. Consider the expression 12 ÷ 3. This is the same as subtracting groups of 3 from 12 until no groups remain. Gather enough 3-blocks, raised sides up, to make a total of 12 units. Begin by subtracting one group of three: 12 – 3= 9. Then, subtract a second group of three from that difference: 9 – 3 = 6. Next, subtract a third group of three: 6– 3 = 3. Finally, subtract a fourth group of three from that difference: 3 – 3 = 0. You can see that when you divide 12 into groups of three, the result is 4 groups.

1. Try to demonstrate $\frac{12}{0}$ as repeated subtraction. Explain why it does not work to model the division expression as repeated subtraction.

Sample Answer:

The expression $\frac{12}{0}$ modeled as repeat subtraction would be 12-0, which equals 12. This does not work because you could go on subtracting zero forever and never arrive at a difference of zero. This is one way to show that a non-zero number divided by zero is undefined.

2. Explain how you could divide 10 water bottles among zero people.

Sample Answer:

It is impossible to divide anything among people when there aren't any people. In the example $\frac{10}{0}$, there is no number that you can multiply by 0 and get a product of 10. Therefore, any non-zero number divided by zero is undefined.

What About Zero Divided by Zero?

Zero divided by zero is essentially asking, "How many zeros are there in zero?"

- Are there no zeros in zero at all?
- Is there exactly one zero in zero?
- Are there many zeros in a zero?

Because these questions cannot be answered definitively, zero divided by zero is undefined (it has no defined value).

When you study more advanced mathematics such as calculus, you may see zero divided by zero referred to as indeterminate, which means that, depending on the circumstances of the problem, it may be defined or may be left undefined.

Numbers Raised to a Power of Zero

What happens when a positive number is raised to a power of zero? A pattern can help you understand this. Begin by evaluating $2^4 = 16$. Continue to evaluate exponents with two as the base, decreasing the power each time.

Example A

Two raised to a power of zero

$2^4 = 16$

÷ 2

$2^3 = 8$

÷ 2

$2^2 = 4$

÷ 2

$2^1 = 2$

÷ 2

$2^0 = 1$

You can see the resulting products are . Look back to and divide each product by the base (2). Observe the pattern of the powers and the products. When you divide each product by 2, the result is the next product. As you continue to work your way down through the pattern, the power in each exponent decreases by one.

When you reach $2^1 = 2$ and divide the product by 2, the result is 1, which is the product of the next exponent in the decreasing sequence. The same result will happen if you try the pattern with other positive non-zero numbers.

Try repeating this activity with the bases 5, 9, and 12 on your own.